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Heat Conduction In A Multilayered Infinitely Long Cylindrical Viscoelastic And/Or Elastic Medium Including Heat Source

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Abstract: **In this work, the problem of heat conduction of an infinitely long cylindrical medium composed of k layers of viscoelastic and/or elastic materials is considered. A distributed heat source acts inside the medium. The lateral surface of the medium is considered stress free and subjected to a thermal shock. The generalized theory of thermoviscoelasticty with one relaxation time is used to model the considered problem. The problem is solved analytically with a direct approach without the customary use of potential functions by using the Laplace transform technique. Numerical inversion of the transformed solution is carried out to obtain the temperature, displacement and stress distributions in a $4-$layered cylinder formed from two different materials. Numerical results are shown visually and discussed.**

Keywords: **Generalized thermoviscoelasticity Heat sources, Multi layers, Thermal shock.**

I. NTRODUCTION

Viscoelastic materials have real-world applications in various technical sectors, including biomaterials, polymers, and the oil exploration sector and nanotechnology, see Svanadze [16]. There is a growing interest in studying viscoelasticity mainly due to the development of new manufacturing techniques, widely used toughened materials such as thermoplastic and toughened epoxy.The isolation of vibration, dampening noise, shock absorption, reliving stress and pain on the human body as well as the protection of sensitive components that are used in different types of machines and/or equipments are some of the main usage of viscoelastic materials.

A material whose stress depends on path and rate of deformation (deformation history) is called viscoelastic material. Because of the usage of this concept, we are able to include viscous fluids, elastic bodies, and elastoplastic materials as special cases within this category.

Before the advent of the field of rheology, the materials were investigated and modeled. At that time, solids were considered as a linear elastic body, also known as a Hookean body, in which the stress is linearly proportional to the strain, while fluids were regarded as Newtonian fluids, or linear viscous fluids, in which the stress is linearly proportional to the strain rate, Cho [4].

In 1784, [5] reported the observation of viscoelastic material behavior. At the mid-nineteenth century, an increasing interest grew for studying solids that did not behave elastically. In 1835, Wilhelm Eduard Weber [19, 20] proposed the first viscoelastic material model, while the first analysis of a polymer (rubber) was proposed by Rudolf Hermann Arndt Kohlrausch was proposed by Rudolf Hermann Arndt[9] using viscoelastic theory. Later on Maxwell [10, 11] introduced the concept of linear viscoelasticity and proposed the following relation

$$\frac{d\sigma}{dt} = E\,\frac{d\varepsilon}{dt} - \frac{\sigma}{\lambda} \qquad (1)$$

where one-dimensional stress is denoted by σ, one-dimensional strain is denoted by ε and the modulus of elasticity and the relaxation time are denoted by E and λ, respectively. This relation can be reduced to the Newtonian model when the relaxation time is taken zero and $\eta = \lambda\, E = const.$, while for infinitely large relaxation time, the Hookean model can be obtained.

The viscoelastic model must be generalized when the temperature varies spatially and/or with time in the object of interest. In such models, a thermoviscoelastic material is defined to be one in which the stress σ_{ij} will depend on the strain history, e_{kl}, and the temperature, T.

The characteristics and behavior of viscoelastic materials have been studied and modeled during several theories. The equations of generalized thermoviscoelasticity have been derived by Sherief et al. [14], and the equations have had their uniqueness and reciprocity theorems proven for them. In the gradient theory of thermoviscoelasticity, where the time derivatives of the strain tensors are a part of the set of independent constitutive variables, Iesan and Quintanilla [8] developed two uniqueness conclusions. Svanadze [17] examined the fundamental boundary value problems (BVPs) of steady vibrations by applying the potential method and the theory of singular integral equations. He did this by considering the linear theory of thermoviscoelasticity for Kelvin-Voigt materials with voids. Iesan [7] was the first person to formulate the fundamental equations of a first strain gradient theory of thermoviscoelasticity of materials with memory, and he also produced a reciprocal theorem in the dynamic theory.

Among the contributions on these theories, Sherief and Allam [13] studied the thermal and mechanical viscoelastic behavior of an elastic material sphere under the influence of a heat source. In addition, they compared between viscoelastic and elastic behaviors. The effect of stochastic thermal inputs on elastic and thermal properties of elastic materials are studied in Allam [1] and Allam et al. [2]. A new generalized fractional thermo-viscoelasticity theory associated with a nonsingular relaxation kernel " Mittag-Leffler relaxation function" is constructed by Sherief et al. [12]. A spatial-temporal fractional order model is proposed to study the dynamic behavior of viscoelastic and thermoelastic nanoplate by Zhao et al. [21]. Tiwari and Abouelregal [18] introduced a modified fractional Kelvin-Voigt model (KVM) to explain the time-dependent behavior of viscoelastic materials based on thermo-viscoelasticity theory and fractional calculus.

The purpose of this work is to investigate the ways in which heat and elastic forces interact inside an annular cylinder that is endlessly long. The cylinder is composed of k layers of viscoelastic materials and/or elastic materials and is considered subjected to a heat source distributed along its axis. The problem is based on thermoviscoelastic theory. Additionally, the purpose of this work is to investigate the transmission of thermal and elastic waves in the presence of a thermal shock. In order to determine the solutions of temperature, displacement, and stress, an analytical method that makes use of the Laplace transform is provided here.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

Let (r, φ, z) represent cylindrical coordinates whose z −axis coincides with the axis of an indefinitely long set of coaxial circular cylinders. This set is composed of k layers of homogeneous isotropic viscoelastic and/or elastic solids having finite conductivities, where the i^{th} layer occupies the region

$$\boldsymbol{r_{i-1} \le r \le r_i} \ \text{ and } \ \boldsymbol{0 \le \varphi \le 2\,\pi} \ \text{ and } \ \boldsymbol{-\infty < z < \infty}\,, \tag{2}$$

where $i = 1,2,\cdots,k$ and r_i is the radius of the i^{th} cylinder, where $r_0 = 0$. The medium's initial state is presumed to be quiescent. In addition to this, we will make the assumption that the lateral surface of the outer cylinder does not experience any traction and is exposed to a temperature distribution that is already known. The governing equations for generalized thermoviscoelasticity theory in the absence of body force and in the existence of a distributed heat source are as follows, see [13]

1. Equation of motion for the i^{th} layer

$$\begin{aligned}\boldsymbol{\rho_i\, \ddot{u}_i} \ &= \int_0^t \left(G_{i1}(t-\tau) + 2\,G_{i2}(t-\tau)\right) \left(\frac{\partial}{\partial \tau}\left(\nabla e_i\right)\right) d\tau \\ &\quad - \int_0^t G_{i2}(t-\tau) \left(\frac{\partial}{\partial \tau}\left(\nabla \wedge \nabla \wedge u_i\right)\right) d\tau - \alpha_i \int_0^t G_i(t-\tau) \left(\frac{\partial}{\partial \tau}\left(\nabla T_i\right)\right) d\tau\,,\end{aligned} \tag{3}$$

where u_i is the displacement vector, $e_i = \nabla . u_i$ is the cubical dilatation, ρ_i is the density, T_i is the absolute temperature, $G_{i1}(t)$ and $G_{i2}(t)$ are relaxation functions, $G_i(t) = 3\, G_{i1}(t) + 2\, G_{i2}(t)$, and α_i denotes the linear thermal expansion coefficient.

2. Heat conduction equation for the i^{th} layer

Without loss of generality, assume that the heat source is distributed along $r \le r_w$.

$$\begin{aligned} \boldsymbol{k_i\,(\nabla^2 T_i)} &= \boldsymbol{\rho_i\, c_i\,(\dot{T}_i + \tau_i\, \ddot{T}_i) - \alpha_i\, T_0 \left[\int_0^t (\dot{e}_i + \tau_i\, \ddot{e}_i)\left(\frac{\partial G_i}{\partial \tau}\right) d\tau - G_i(0)\,(\dot{e}_i + \tau_i\, \ddot{e}_i)\right]} \\ &\quad \boldsymbol{-\rho_i} \begin{cases} \boldsymbol{(Q + \tau_i\, \dot{Q})} & , \text{ if } \quad \boldsymbol{0 \le r \le r_w\,, w \in \{1, 2, \cdots, k-1\}} \\ \boldsymbol{0} & , \text{ if } \quad \boldsymbol{r_w < r \le r_k} \end{cases} \end{aligned} \tag{4}$$

where Q refers to the quantity of heat that was produced by the heat source, τ_i, a constant with dimension of time, is the thermal relaxation time, c_i represents the specific heat produced at constant strain, k_i is the thermal conductivity of the medium and T_0 represents the reference temperature, which is assumed to be

$|(T_i - T_0)/T_0| \ll 1.$

3. Constitutive equations for the i^{th} layer

$$\boldsymbol{\sigma_i = 2 \int_0^t G_{i2}(t-\tau)\left(\frac{\partial e_i}{\partial \tau}\right) d\tau + \mathrm{I}\left(\int_0^t G_{i1}(t-\tau)\left(\frac{\partial e_i}{\partial \tau}\right) d\tau - \alpha_i \int_0^t G_i(t-\tau)\left(\frac{\partial T_i}{\partial \tau}\right) d\tau\right)}, \tag{5}$$

where I denotes the identity matrix, σ_i refers to the stress tensor. Moreover, the strain tensor, $\mathbf{e}_i$, is given by

$$\mathbf{e}_i = \frac{1}{2}(\boldsymbol{\nabla u_i} + (\boldsymbol{\nabla u_i}))', \tag{6}$$

where the matrix transpose of the matrix A is denoted by A'.

Within the scope of this study, we examine the problem in two dimensions: r and φ. In this case, the displacement vector can be shown to have the following components

$$\boldsymbol{u_{ir} = u_i(r, \varphi, t)}\,, \quad \boldsymbol{u_{i\varphi} = v_i(r, \varphi, t)} \quad \text{and} \quad \boldsymbol{u_{iz} = 0}\,. \tag{7}$$

Based on that, the strain components can be derived as

$$\mathbf{e}_i = \begin{bmatrix} \boldsymbol{\frac{\partial u_i}{\partial r}} & \boldsymbol{\frac{1}{2}\left(\frac{1}{r}\frac{\partial u_i}{\partial \varphi} + \frac{\partial v_i}{\partial r} - \frac{v_i}{r}\right)} & \boldsymbol{0} \\ \boldsymbol{\frac{1}{2}\left(\frac{1}{r}\frac{\partial u_i}{\partial \varphi} + \frac{\partial v_i}{\partial r} - \frac{v_i}{r}\right)} & \boldsymbol{\frac{1}{r}\left(u_i + \frac{\partial v_i}{\partial \varphi}\right)} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \tag{8}$$

This allows us to derive the following expression for the components of the thermoelastic stress tensor σ_i:

$$\boldsymbol{\sigma_{irr} = 2 \int_0^t G_{i2}(t-\tau)\left(\frac{\partial}{\partial \tau}\left(\frac{\partial u_i}{\partial r}\right)\right) d\tau + \int_0^t G_{i1}(t-\tau)\left(\frac{\partial e_i}{\partial \tau}\right) d\tau - \alpha_i \int_0^t G_i(t-\tau)\left(\frac{\partial T_i}{\partial \tau}\right) d\tau}\,, \tag{9}$$

$$\begin{aligned} \boldsymbol{\sigma_{i\varphi\varphi}} &= \boldsymbol{2 \int_0^t G_{i2}(t-\tau)\left(\frac{\partial}{\partial \tau}\left(\frac{1}{r}\left(u_i + \frac{\partial v_i}{\partial \varphi}\right)\right)\right) d\tau + \int_0^t G_{i1}(t-\tau)\left(\frac{\partial e_i}{\partial \tau}\right) d\tau} \\ &\quad \boldsymbol{-\alpha_i \int_0^t G_i(t-\tau)\left(\frac{\partial T_i}{\partial \tau}\right) d\tau}\,, \end{aligned} \tag{10}$$

$$\boldsymbol{\sigma_{izz} = \int_0^t G_{i1}(t-\tau)\left(\frac{\partial e_i}{\partial \tau}\right) d\tau - \alpha_i \int_0^t G_i(t-\tau)\left(\frac{\partial T_i}{\partial \tau}\right) d\tau}\,, \tag{11}$$

$$\boldsymbol{\sigma_{ir\varphi} = \int_0^t G_{i2}(t-\tau)\left(\frac{\partial}{\partial \tau}\left(\left(\frac{\partial v_i}{\partial r} + \frac{1}{r}\left(\frac{\partial u_i}{\partial \varphi} - v_i\right)\right)\right)\right) d\tau}\,, \tag{12}$$

$$\boldsymbol{\sigma_{iz\varphi} = \sigma_{irz} = 0}\,. \tag{13}$$

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The equation of motion (3) has the following two components

$$\rho_i \frac{\partial^2 u_i}{\partial t^2} = \int_0^t \left(G_{i1}(t-\tau) + 2\, G_{i2}(t-\tau)\right) \left(\frac{\partial}{\partial \tau}\left(\frac{\partial e_i}{\partial r}\right)\right) d\tau - \alpha_i \int_0^t G_i(t-\tau) \left(\frac{\partial}{\partial \tau}\left(\frac{\partial T_i}{\partial r}\right)\right) d\tau - \int_0^t G_{i2}(t-\tau) \left(\frac{\partial}{\partial \tau}\left(\frac{1}{r}\frac{\partial}{\partial \varphi}\left(\frac{1}{r}\left(\frac{\partial (r\, v_i)}{\partial r} - \frac{\partial u_i}{\partial \varphi}\right)\right)\right)\right) d\tau\,, \tag{14}$$

$$\rho_i \frac{\partial^2 v_i}{\partial t^2} = \int_0^t \left(G_{i1}(t-\tau) + 2\, G_{i2}(t-\tau)\right) \left(\frac{\partial}{\partial \tau}\left(\frac{1}{r}\frac{\partial e_i}{\partial \varphi}\right)\right) d\tau - \alpha_i \int_0^t G_i(t-\tau) \left(\frac{\partial}{\partial \tau}\left(\frac{1}{r}\frac{\partial T_i}{\partial \varphi}\right)\right) d\tau + \int_0^t G_{i2}(t-\tau) \left(\frac{\partial}{\partial \tau}\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\left(\frac{\partial (r\, v_i)}{\partial r} - \frac{\partial u_i}{\partial \varphi}\right)\right)\right)\right) d\tau\,. \tag{15}$$

Additionally, the cubical dilatation e_i is obtained as

$$e_i = \frac{1}{r}\left(\frac{\partial (r\, u_i)}{\partial r} + \frac{\partial v_i}{\partial \varphi}\right). \tag{16}$$

In order to simplify the equations and make the calculations easier to understand by decreasing the number of parameters and to give a better insight of the nature of the problem and the controlling parameters, we introduce the following dimensionless variables

$$r^* = V\,\eta\, r\,, \quad r_i^* = \frac{r_i}{r_n}\,, \quad u_i^* = V\,\eta\, u_i\,, \quad v_i^* = V\,\eta\, v_i\,, \quad t^* = V^2\,\eta\, t\,, \quad \tau_i^* = V^2\,\eta\,\tau_i\,,$$

$$\theta_i^* = \alpha_n\,(T_i - T_0)\,, \quad \sigma_i^* = \frac{\sigma_i}{\lambda_n + 2\,\mu_n}\,, \quad G_{i1}^* = \frac{G_{i1}}{\lambda_n + 2\,\mu_n}\,, \quad G_{i2}^* = \frac{G_{i2}}{\lambda_n + 2\,\mu_n}\,, Q^* = \frac{\alpha_n\,\rho_n\, Q}{k_n\, V^2\,\eta^2}$$

where λ_i is the Lame''s constant, μ_i denotes the modulus of rigidity, $V = \sqrt{(\lambda_n + 2\,\mu_n)/\rho_n}$ represents the speed at which longitudinal isothermal waves travel across space and $\eta = \rho_n\, c_n / k_n$. Rewriting the energy equation (4) and the equation of motion components (14) and (15) using the dimensionless variables we get

$$\nabla^2\,\theta_i = \eta_i\,(\dot{\theta}_i + \tau_i\,\ddot{\theta}_i) - \varepsilon_i\left[\int_0^t (\dot{e}_i + \tau_i\,\ddot{e}_i)\left(\frac{\partial G_i(\tau)}{\partial \tau}\right) d\tau - G_i(0)\,(\dot{e}_i + \tau_i\,\ddot{e}_i)\right] - \xi_i \begin{cases} (Q + \tau_i\,\dot{Q}) & , \text{ if } \quad 0 \le r \le r_w\,, w \in \{1, 2, \cdots, k-1\} \\ 0 & , \text{ if } \quad r_w < r \le r_k \end{cases}, \tag{17}$$

$$\psi_i \frac{\partial^2 u_i}{\partial t^2} = \int_0^t \left(G_{i1}(t-\tau) + 2\, G_{i2}(t-\tau)\right) \left(\frac{\partial}{\partial \tau}\left(\frac{\partial e_i}{\partial r}\right)\right) d\tau - \delta_i \int_0^t G_i(t-\tau) \left(\frac{\partial}{\partial \tau}\left(\frac{\partial \theta_i}{\partial r}\right)\right) d\tau - \int_0^t G_{i2}(t-\tau) \left(\frac{\partial}{\partial \tau}\left(\frac{1}{r}\frac{\partial}{\partial \varphi}\left(\frac{1}{r}\left(\frac{\partial (r\, v_i)}{\partial r} - \frac{\partial u_i}{\partial \varphi}\right)\right)\right)\right) d\tau\,, \tag{18}$$

$$\psi_i \frac{\partial^2 v_i}{\partial t^2} = \int_0^t \left(G_{i1}(t-\tau) + 2\, G_{i2}(t-\tau)\right) \left(\frac{\partial}{\partial \tau}\left(\frac{1}{r}\frac{\partial e_i}{\partial \varphi}\right)\right) d\tau - \delta_i \int_0^t G_i(t-\tau) \left(\frac{\partial}{\partial \tau}\left(\frac{1}{r}\frac{\partial \theta_i}{\partial \varphi}\right)\right) d\tau + \int_0^t G_{i2}(t-\tau) \left(\frac{\partial}{\partial \tau}\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\left(\frac{\partial (r\, v_i)}{\partial r} - \frac{\partial u_i}{\partial \varphi}\right)\right)\right)\right) d\tau\,. \tag{19}$$

where

$$\eta_i = \frac{\rho_i\, c_i\, k_n}{\rho_n\, c_n\, k_i}\,, \quad \varepsilon_i = \frac{(\lambda_n + 2\,\mu_n)\,\alpha_i\,\alpha_n\, k_n\, T_0}{k_i\,\rho_n\, c_n}\,, \quad \xi_i = \frac{\rho_i\, k_n}{\rho_n\, k_i}\,, \quad \psi_i = \frac{\rho_i}{\rho_n}\,, \quad \delta_i = \frac{\alpha_i}{\alpha_n}$$

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The expression for the dimensionless stress components can be written as

$$\sigma_{irr} = 2 \int_0^t G_{i2}(t-\tau)\left(\frac{\partial}{\partial \tau}\left(\frac{\partial u_i}{\partial r}\right)\right) d\tau + \int_0^t G_{i1}(t-\tau)\left(\frac{\partial e_i}{\partial \tau}\right) d\tau - \delta_i \int_0^t G_i(t-\tau)\left(\frac{\partial \theta_i}{\partial \tau}\right) d\tau , \quad (20)$$

$$\sigma_{i\varphi\varphi} = 2 \int_0^t G_{i2}(t-\tau)\left(\frac{\partial}{\partial \tau}\left(\frac{1}{r}\left(u_i + \frac{\partial v_i}{\partial \varphi}\right)\right)\right) d\tau + \int_0^t G_{i1}(t-\tau)\left(\frac{\partial e_i}{\partial \tau}\right) d\tau - \delta_i \int_0^t G_i(t-\tau)\left(\frac{\partial \theta_i}{\partial \tau}\right) d\tau , \quad 21)$$

$$\sigma_{izz} = \int_0^t G_{i1}(t-\tau)\left(\frac{\partial e_i}{\partial \tau}\right) d\tau - \delta_i \int_0^t G_i(t-\tau)\left(\frac{\partial \theta_i}{\partial \tau}\right) d\tau , \quad (22)$$

$$\sigma_{ir\varphi} = \int_0^t G_{i2}(t-\tau)\left(\frac{\partial}{\partial \tau}\left(\left(\frac{\partial v_i}{\partial r} + \frac{1}{r}\left(\frac{\partial u_i}{\partial \varphi} - v_i\right)\right)\right)\right) d\tau , \quad (23)$$

It is presumed that the problem's initial conditions are homogeneous. In addition, the dimensionless boundary conditions listed below will be applied.

1. On the outer surface $(r = r_k)$

(a) The surface of the cylinder has no traction (Mechanical condition).

$$\sigma_{krr}(r_k, \varphi, t) = \sigma_{kr\varphi}(r_k, \varphi, t) = 0 \quad \text{for} \quad t \geq 0 . \quad (24)$$

(b) The cylinder's surface is heated to a known temperature $F(\varphi, t)$ (Thermal condition).

$$\theta_k(r_k, \varphi, t) = F(\varphi, t) \quad \text{for} \quad t \geq 0 , \quad (25)$$

2. At the interface between two consecutive layers $(r = r_{i-1},\ i = 2,3,4,\cdots,k)$, the following continuity conditions must be satisfied for $t \geq 0$

$$\theta_i(r_{i-1}, \varphi, t) = \theta_{(i-1)}(r_{i-1}, \varphi, t) \quad \text{and} \quad q_{ir}(r_{i-1}, \varphi, t) = q_{(i-1)r}(r_{i-1}, \varphi, t). \quad (26)$$

$$u_i(r_{i-1}, \varphi, t) = u_{(i-1)}(r_{i-1}, \varphi, t) \quad \text{and} \quad v_i(r_{i-1}, \varphi, t) = v_{(i-1)}(r_{i-1}, \varphi, t). \quad (27)$$

$$\sigma_{irr}(r_{i-1}, \varphi, t) = \sigma_{(i-1)rr}(r_{i-1}, \varphi, t) \quad \text{and} \quad \sigma_{ir\varphi}(r_{i-1}, \varphi, t) = \sigma_{(i-1)r\varphi}(r_{i-1}, \varphi, t). \quad (28)$$

III. SOLUTION OF THE PROBLEM IN THE LAPLACE TRANSFORM DOMAIN

In the following, we are going to state our problem and then find a solution to it in the Laplace transform domain. For a given function $g(r, \varphi, t)$ defined for $t \geq 0$, the Laplace transform is defined by

$$\bar{g}(r, \varphi, s) = \mathcal{L}\{g(r, \varphi, t)\} = \int_0^\infty e^{-s\,t}\, g(r, \varphi, t)\, dt ,$$

with its inverse transform

$$g(r, \varphi, t) = \mathcal{L}^{-1}\{\bar{g}(r, \varphi, s)\} = \frac{1}{2\pi j}\int_{\gamma - j\,\infty}^{\gamma + j\,\infty} e^{st}\, \bar{g}(r, \varphi, s)\, ds , \quad j = \sqrt{-1} ,$$

Upon applying this definition to (17) - (19) with the usage of the homogeneous initial conditions, we get

$$\left(\nabla^2 - \eta_i\, s\,(1 + \tau_i\, s)\right)\bar{\theta}_i = \varepsilon_i\, s\,(1 + \tau_i\, s)\left(2\, G_i(0) - s\,\bar{G}_i(s)\right)\bar{e}_i - \xi_i \begin{cases} (1 + \tau_i\, s)\,\bar{Q} & , \text{ if } \quad 0 \leq r \leq r_w, w \in \{1, 2, \cdots, k-1\} \\ 0 & , \text{ if } \quad r_w < r \leq r_k \end{cases} , \quad (29)$$

$$\psi_i\, s\, \bar{u}_i = (\bar{G}_{i1}(s) + 2\,\bar{G}_{i2}(s))\left(\frac{\partial \bar{e}_i}{\partial r}\right) - \delta_i\, \bar{G}_i(s)\left(\frac{\partial \bar{\theta}_i}{\partial r}\right) - \bar{G}_{i2}(s)\left(\frac{1}{r}\frac{\partial}{\partial \varphi}\left(\frac{1}{r}\left(\frac{\partial (r\,\bar{v}_i)}{\partial r} - \frac{\partial \bar{u}_i}{\partial \varphi}\right)\right)\right) , \quad (30)$$

$$\psi_i\, s\, \bar{v}_i = (\bar{G}_{i1}(s) + 2\,\bar{G}_{i2}(s))\left(\frac{1}{r}\frac{\partial \bar{e}_i}{\partial \varphi}\right) - \delta_i\, \bar{G}_i(s)\left(\frac{1}{r}\frac{\partial \bar{\theta}_i}{\partial \varphi}\right) + \bar{G}_{i2}(s)\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\left(\frac{\partial (r\,\bar{v}_i)}{\partial r} - \frac{\partial \bar{u}_i}{\partial \varphi}\right)\right)\right). \quad (31)$$

By acting $\frac{1}{r}\frac{\partial}{\partial r}(r)$ to (30) and $\frac{1}{r}\frac{\partial}{\partial \varphi}$ to (31) and then add, we get

$$((\bar{G}_{i1}(s) + 2\,\bar{G}_{i2}(s))\,\nabla^2 - \psi_i\, s)\,\bar{e}_i - \delta_i\,\bar{G}_i(s)\,\nabla^2\,\bar{\theta}_i = 0\,. \tag{32}$$

By removing the transformed strain $\bar{e}_i(r,\varphi,s)$ from (29) and (32), we are able to derive the transformed temperature by solving the following fourth-order partial differential equation.

$$\Big\{(\bar{G}_{i1} + 2\,\bar{G}_{i2})\,\nabla^4 - s\,\Big(\psi_i + (1+\tau_i\, s)\,\big(\eta_i\,(\bar{G}_{i1} + 2\,\bar{G}_{i2}) + \delta_i\,\varepsilon_i\,\bar{G}_i\,(2\,G_i(0) - s\,\bar{G}_i)\big)\Big)\,\nabla^2 + \psi_i\,\eta_i\,s^2\,(1 + \tau_i\, s)\Big\}\,\bar{\theta}_i = \psi_i\,\xi_i \begin{cases} s\,(1+\tau_i\, s)\,\bar{Q} & , \ if \quad 0 \le r \le r_w \\ 0 & , \ if \quad r_w < r \le r_k \end{cases}, \tag{33}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2}\,. \tag{34}$$

Rewriting (33) as

$$(\nabla^2 - k_{i1}^2)(\nabla^2 - k_{i2}^2)\,\bar{\theta}_i(r,\varphi,s) = \psi_i\,\xi_i \begin{cases} s\,(1+\tau_i\, s)\,\bar{Q} & , \ if \quad 0 \le r \le r_w \\ 0 & , \ if \quad r_w < r \le r_k \end{cases}, \tag{35}$$

we have a solution for $\bar{\theta}_i(r,\varphi,s)$ given by

$$\bar{\theta}_i(r,\varphi,s) = \sum_{m=1}^{2}\,\bar{\theta}_{im}(r,\varphi,s) + \frac{\gamma_i\, Q_0}{s^2} \begin{cases} 1 & , \ if \quad 0 \le r \le r_w \\ 0 & , \ if \quad r_w < r \le r_k \end{cases}, \tag{36}$$

where $\gamma_i = c_n/c_i$, $Q = Q_0\,H(t)$, Q_0 is the strength of the induced heat source, $\bar{\theta}_{im}(r,\varphi,s)$ is the solution of $(\nabla^2 - k_{im}^2)\,\bar{\theta}_{im}(r,\varphi,s) = 0$ and k_{im}^2, $m = 1,2$ are the roots of

$$(\bar{G}_{i1} + 2\,\bar{G}_{i2})\,K^4 - s\,(\psi_i + (1+\tau_i\, s)\,(\eta_i\,(\bar{G}_{i1} + 2\,\bar{G}_{i2}) + \delta_i\,\varepsilon_i\,\bar{G}_i\,(2\,G_i(0) - s\,\bar{G}_i)))\,K^2 + \psi_i\,\eta_i\,s^2\,(1+\tau_i\, s) = 0\,, \tag{37}$$

After performing the necessary operations on (35), we are able to get the solution in the form of

$$\bar{\theta}_i(r,\varphi,s) = \sum_{n=0}^{\infty}\,\Big(\sum_{m=1}^{2}\,((\bar{G}_{i1} + 2\,\bar{G}_{i2})\,k_{im}^2 - \psi_i\, s\,)\,\times(\,\bar{A}_{imn}(s)\,I_n(k_{im}\, r) + \bar{B}_{imn}(s)\,K_n(k_{im}\, r)))\,\cos(n\,\varphi) + \frac{\gamma_i\, Q_0}{s^2} \begin{cases} 1 & , \ \text{for} \quad i = 1,2,3,\cdots,w \\ 0 & , \ \text{for} \quad i = w+1, w+2,\cdots,k \end{cases}, \tag{38}$$

where $\bar{A}_{imn}(s)$ and $\bar{B}_{imn}(s)$ depend on s and I_n and K_n are the modified Bessel functions of the first and second kinds of order n, respectively. Similarly, we can show that $\bar{e}_i(r,\varphi,s)$ is a solution of (33) with **RHS** equal zero. Hence, we have

$$\bar{e}_i(r,\varphi,s) = \delta_i\,\bar{G}_i(s)\,\sum_{n=0}^{\infty}\,\Big(\sum_{m=1}^{2}\,k_{im}^2\,(\bar{A}_{imn}(s)\,I_n(k_{im}\, r) + \bar{B}_{imn}(s)\,K_n(k_{im}\, r))\Big)\,\cos(n\,\varphi)\,, \tag{39}$$

Combining equation (30) and the Laplace transform of (16) yields

$$(\bar{G}_{i2}\,\nabla^2 - \psi_i\, s)\,(r\,\bar{u}_i) = \delta_i\,\bar{G}_i\, r\,\frac{\partial \bar{\vartheta}_i}{\partial r} + 2\,\bar{G}_{i2}\,\bar{e}_i - (\bar{G}_{i1} + \bar{G}_{i2})\, r\,\frac{\partial \bar{e}_i}{\partial r}\,. \tag{40}$$

Recall (38) and (39) in the **RHS** of (40), we get

$$\left(\nabla^2 - \frac{\psi_i\, s}{\bar{G}_{i2}}\right)(r\,\bar{u}_i) = \delta_i\,\bar{G}_i\,\sum_{n=0}^{\infty}\,\left(\sum_{m=1}^{2}\left[\bar{A}_{imn}\left(\left(k_{im}^2\,(n+2) - \frac{n\,\psi_i\, s}{\bar{G}_{i2}}\right) I_n(k_{im}\, r) + k_{im}\, r\,\left(k_{im}^2 - \frac{\psi_i\, s}{\bar{G}_{i2}}\right) I_{n+1}(k_{im}\, r)\right) + \bar{B}_{imn}\left(\left(k_{im}^2\,(n+2) - \frac{n\,\psi_i\, s}{\bar{G}_{i2}}\right) K_n(k_{im}\, r) - k_{im}\, r\,\left(k_{im}^2 - \frac{\psi_i\, s}{\bar{G}_{i2}}\right) K_{n+1}(k_{im}\, r)\right)\right]\right)\cos(n\,\varphi)\,, \tag{42}$$

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which has a solution in the form

$$r\,\bar{u}_i(r,\varphi,s)=\delta_i\,\bar{G}_i\sum_{n=0}^{\infty}\Big(\sum_{m=1}^{2}\big[\bar{A}_{imn}\big(n\,I_n(k_{im}\,r)+k_{im}\,r\,I_{n+1}(k_{im}\,r)\big)+\bar{B}_{imn}\big(n\,K_n(k_{im}\,r)-k_{im}\,r\,K_{n+1}(k_{im}\,r)\big)\big]\Big)\cos(n\,\varphi)+\sum_{n=1}^{\infty}\left(\bar{C}_{in}(s)\,I_n\left(\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r\right)+\bar{D}_{in}(s)\,K_n\left(\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r\right)\right)\cos(n\,\varphi) \tag{43}$$

where $\bar{C}_{in}(s)$ and $\bar{D}_{in}(s)$ depend on s. Apply Laplace transform to (16), we get

$$\frac{\partial\,\bar{v}_i}{\partial\varphi}=r\,\bar{e}_i-\frac{\partial(r\,\bar{u}_i)}{\partial r}. \tag{44}$$

Using (39) and (43) in the **RHS** of the previous equation, the solution of $\bar{v}_i(r,\varphi,s)$ can be obtained as

$$\bar{v}_i(r,\varphi,s)=-\,\delta_i\,\bar{G}_i\sum_{n=0}^{\infty}\left(\sum_{m=1}^{2}\frac{n}{r}\left(\bar{A}_{imn}\,I_n(k_{im}\,r)+\bar{B}_{imn}\,K_n(k_{im}\,r)\right)\right)\sin(n\,\varphi)-\sum_{n=1}^{\infty}\left[\bar{C}_{in}\left(\frac{1}{r}\,I_n\left(\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r\right)+\frac{1}{n}\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\,I_{n+1}\left(\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r\right)\right)\bar{D}_{in}\left(\frac{1}{r}\,K_n\left(\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r\right)-\frac{1}{n}\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\,K_{n+1}\left(\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r\right)\right)\right]\sin(n\,\varphi) \tag{45}$$

To get the stress components, we need to make a substitution from (38), (39), (43) and (45) in (20) - (23), and then carry out the appropriate procedures. This result in

$$\bar{\sigma}_{irr}(r,\varphi,s)=\frac{2\,\delta_i\,s\,\bar{G}_{i2}\,\bar{G}_i}{r^2}\sum_{n=0}^{\infty}\left(\sum_{m=1}^{2}\left[\bar{A}_{imn}\left(\left(n(n-1)+\frac{r^2\,\psi_i\,s}{2\,\bar{G}_{i2}}\right)I_n(k_{im}\,r)-k_{im}\,r\,I_{n+1}(k_{im}\,r)\right)+\bar{B}_{imn}\left(\left(n(n-1)+\frac{r^2\,\psi_i\,s}{2\,\bar{G}_{i2}}\right)K_n(k_{im}\,r)+k_{im}\,r\,K_{n+1}(k_{im}\,r)\right)\right]\right)\cos(n\,\varphi)+\frac{2\,s\,\bar{G}_{i2}}{r^2}\sum_{n=1}^{\infty}\left[\bar{C}_{in}\left((n-1)\,I_n\left(\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r\right)+\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r\,I_{n+1}\left(\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r\right)\right)+\bar{D}_{in}\left((n-1)\,K_n\left(\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r\right)-\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r\,K_{n+1}\left(\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r\right)\right)\right]\cos(n\,\varphi)-\frac{\delta_i\,\gamma_i\,\bar{G}_i\,Q_0}{s}\begin{cases}1 & ,\ \text{for}\quad i=1,2,3,\cdots,w\\ 0 & ,\ \text{for}\quad i=w+1,w+2,\cdots,k\end{cases} \tag{46}$$

$$\bar{\sigma}_{ir\varphi}(r,\varphi,s)=-\,\frac{2\,\delta_i\,s\,\bar{G}_{i2}\,\bar{G}_i}{r^2}\sum_{n=0}^{\infty}\left(\sum_{m=1}^{2}\left[\bar{A}_{imn}\,n\left((n-1)\,I_n(k_{im}\,r)+k_{im}\,r\,I_{n+1}(k_{im}\,r)\right)+\bar{B}_{imn}\,n\left((n-1)\,K_n(k_{im}\,r)-k_{im}\,r\,K_{n+1}(k_{im}\,r)\right)\right]\right)\sin(n\,\varphi)-\frac{2\,s\,\bar{G}_{i2}}{r^2}\sum_{n=1}^{\infty}\frac{1}{n}\left[\bar{C}_{in}\left(\left(n\,(n-1)+\frac{r^2\,\psi_i\,s}{2\,\bar{G}_{i2}}\right)I_n\left(\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r\right)-r\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\,I_{n+1}\left(\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r\right)\right)+\bar{D}_{in}\left(\left(n\,(n-1)+\frac{r^2\,\psi_i\,s}{2\,\bar{G}_{i2}}\right)K_n\left(\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r\right)+r\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\,K_{n+1}\left(\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r\right)\right)\right]\sin(n\,\varphi)\,. \tag{47}$$

Since we are seeking a bounded solution for all the physical properties, therefore we will neglected all terms containing Bessel functions of the second kind for all solution in the innermost layer ($r\le r_1$).

In order to determine the unknown parameters $\bar{A}_{imn}(s)$, $\bar{B}_{imn}(s)$, $\bar{C}_{in}(s)$ and $\bar{D}_{in}(s)$, we need to apply the boundary conditions. By expand $F(\varphi,t)$ in its Fourier cosine series (even periodic extension), we are able to reach the following result:

$$\bar{F}(\varphi,s)=\sum_{n=0}^{\infty}\bar{f}_n(s)\cos(n\,\varphi)\,, \tag{48}$$

where $\bar{f}_n(s)$ represents the Fourier coefficients of $\bar{F}(\varphi, s)$ across the interval $[\,0, 2\,\pi\,]$. When the Laplace transform is applied on (25), and then $\bar{\theta}_x(r_k, \varphi, s)$ and $\bar{F}(\varphi, s)$ is substituted from (38) and (48), respectively, we get the following result after equating the coefficients of $\cos(n\,\varphi)$ on both sides:

$$\sum_{m=1}^{2} \left((\bar{G}_{k1} + 2\,\bar{G}_{k2})\,k_{km}^2 - \psi_k\,s \right) \left(\bar{A}_{kmn}(s)\,I_n(k_{km}\,r_k) + \bar{B}_{kmn}(s)\,K_n(k_{km}\,r_k) \right) = \bar{f}_n(s)\,, \quad n \geq 0\,. \tag{49}$$

Similarly, applying Laplace transform to (24) and using (46) and (47) give

$$\sum_{m=1}^{2} \left[\bar{A}_{km0} \left(\frac{r_k^2\,\psi_k\,s}{2\,\bar{G}_{k2}}\, I_0(k_{km}\,r_k) - k_{km}\,r_k\,I_1(k_{km}\,r_k) \right) + \bar{B}_{km0} \left(\frac{r_k^2\,\psi_k\,s}{2\,\bar{G}_{k2}}\, K_0(k_{km}\,r_k) + k_{km}\,r_k\,K_1(k_{km}\,r_k) \right) \right] = 0\,, \tag{50}$$

$$\delta_k\,\bar{G}_k \sum_{m=1}^{2} \left[\bar{A}_{kmn} \left(\left(n(n-1) + \frac{r_k^2\,\psi_k\,s}{2\,\bar{G}_{k2}} \right) I_n(k_{km}\,r_k) - k_{km}\,r_k\,I_{n+1}(k_{km}\,r_k) \right) + \bar{B}_{kmn} \left(\left(n(n-1) + \frac{r_k^2\,\psi_k\,s}{2\,\bar{G}_{k2}} \right) K_n(k_{km}\,r_k) + k_{km}\,r_k\,K_{n+1}(k_{km}\,r_k) \right) \right] + \bar{C}_{kn} \left((n-1)\,I_n\left(\sqrt{\frac{\psi_k\,s}{\bar{G}_{k2}}}\;r_k \right) + \sqrt{\frac{\psi_k\,s}{\bar{G}_{k2}}}\;r_k\,I_{n+1}\left(\sqrt{\frac{\psi_k\,s}{\bar{G}_{k2}}}\;r_k \right) \right) + \bar{D}_{kn} \left((n-1)\,K_n\left(\sqrt{\frac{\psi_k\,s}{\bar{G}_{k2}}}\;r_k \right) - \sqrt{\frac{\psi_k\,s}{\bar{G}_{k2}}}\;r_k\,K_{n+1}\left(\sqrt{\frac{\psi_k\,s}{\bar{G}_{k2}}}\;r_k \right) \right) = 0\,, \quad n \geq 1\,, \tag{51}$$

$$\delta_k\,\bar{G}_k \sum_{m=1}^{2} \left[\bar{A}_{kmn}\,n \left((n-1)\,I_n(k_{km}\,r_k) + k_{km}\,r_k\,I_{n+1}(k_{km}\,r_k) \right) + \bar{B}_{kmn}\,n \left((n-1)\,K_n(k_{km}\,r_k) - k_{km}\,r_k\,K_{n+1}(k_{km}\,r_k) \right) \right] + \frac{1}{n} \left[\bar{C}_{kn} \left(\left(n\,(n-1) + \frac{r_k^2\,\psi_k\,s}{2\,\bar{G}_{k2}} \right) I_n\left(\sqrt{\frac{\psi_k\,s}{\bar{G}_{k2}}}\;r_k \right) - \sqrt{\frac{\psi_k\,s}{\bar{G}_{k2}}}\;r_k\;I_{n+1}\left(\sqrt{\frac{\psi_k\,s}{\bar{G}_{k2}}}\;r_k \right) \right) + \bar{D}_{kn} \left(\left(n\,(n-1) + \frac{r_k^2\,\psi_k\,s}{2\,\bar{G}_{k2}} \right) K_n\left(\sqrt{\frac{\psi_k\,s}{\bar{G}_{k2}}}\;r_k \right) + \sqrt{\frac{\psi_k\,s}{\bar{G}_{k2}}}\;r_k\;K_{n+1}\left(\sqrt{\frac{\psi_k\,s}{\bar{G}_{k2}}}\;r_k \right) \right) \right] = 0\,, \quad n \geq 1\,, \tag{52}$$

Perform the continuity equations (26) - (28), we get

$$\sum_{m=1}^{2} \left[\left((\bar{G}_{i1} + 2\,\bar{G}_{i2})\,k_{im}^2 - \psi_i\,s \right) \left(\bar{A}_{imn}(s)\,I_n(k_{im}\,r_{i-1}) + \bar{B}_{imn}(s)\,K_n(k_{im}\,r_{i-1}) \right) - \left((\bar{G}_{(i-1)1} + 2\,\bar{G}_{(i-1)2})\,k_{(i-1)m}^2 - \psi_{(i-1)}\,s \right) \times \left(\bar{A}_{(i-1)mn}(s)\,I_n\left(k_{(i-1)m}\,r_{i-1}\right) + \bar{B}_{(i-1)mn}(s)\,K_n\left(k_{(i-1)m}\,r_{i-1}\right) \right) \right] = \frac{Q_0}{s^2} \begin{cases} (\gamma_{i-1} - \gamma_i) & , \text{ for } \quad i = 2, 3, \cdots, w \\ \gamma_{i-1} & , \text{ for } \quad i = w+1 \\ 0 & , \text{ for } \quad i = w+2, w+3, \cdots, k \end{cases} . \tag{53}$$

$$\sum_{m=1}^{2} \left[\left((\bar{G}_{i1} + 2\,\bar{G}_{i2})\,k_{im}^2 - \psi_i\,s \right) \left(\bar{A}_{imn}(s)\,I_n(k_{im}\,r_{i-1}) + \bar{B}_{imn}(s)\,K_n(k_{im}\,r_{i-1}) \right) - \left((\bar{G}_{(i-1)1} + 2\,\bar{G}_{(i-1)2})\,k_{(i-1)m}^2 - \psi_{(i-1)}\,s \right) \times \left(\bar{A}_{(i-1)mn}(s)\,I_n\left(k_{(i-1)m}\,r_{i-1}\right) + \bar{B}_{(i-1)mn}(s)\,K_n\left(k_{(i-1)m}\,r_{i-1}\right) \right) \right] = 0\,, \quad n \geq 1\,. \tag{54}$$

$$\sum_{m=1}^{2} \left[\frac{1}{1+\tau_i\,s} \left((\bar{G}_{i1} + 2\,\bar{G}_{i2})\,k_{im}^2 - \psi_i\,s \right) \left(\bar{A}_{imn} \left(\frac{n}{r_{i-1}}\,I_n(k_{im}\,r_{i-1}) + k_{im}\,I_{n+1}(k_{im}\,r_{i-1}) \right) + \bar{B}_{imn} \left(\frac{n}{r_{i-1}}\,K_n(k_{im}\,r_{i-1}) - k_{im}\,K_{n+1}(k_{im}\,r_{i-1}) \right) \right) - \frac{1}{1+\tau_{i-1}\,s} \left((\bar{G}_{(i-1)1} + 2\,\bar{G}_{(i-1)2})\,k_{(i-1)m}^2 - \psi_{i-1}\,s \right) \times \left(\bar{A}_{(i-1)mn} \left(\frac{n}{r_{i-1}}\,I_n\left(k_{(i-1)m}\,r_{i-1}\right) + k_{(i-1)m}\,I_{n+1}\left(k_{(i-1)m}\,r_{i-1}\right) \right) + \bar{B}_{(i-1)mn} \left(\frac{n}{r_{i-1}}\,K_n\left(k_{(i-1)m}\,r_{i-1}\right) - \right.\right.\right.$$

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$$k_{(i-1)m}\, K_{n+1}\big(k_{(i-1)m}\, r_{i-1}\big)\Big)\Big)\Big] = 0\,, n \geq 0\,, \quad (55)$$

$$\sum_{m=1}^{2}\Big[\delta_i\, \bar{G}_i\,\big(\bar{A}_{im0}\, k_{im}\, r_{i-1}\, I_1(k_{im}\, r_{i-1}) - \bar{B}_{im0}\, k_{im}\, r_{i-1}\, K_1(k_{im}\, r_{i-1})\big) - \delta_{i-1}\, \bar{G}_{i-1}\,\Big(\bar{A}_{(i-1)m0}\, k_{(i-1)m}\, r_{i-1}\, I_1\big(k_{(i-1)m}\, r_{i-1}\big) - \bar{B}_{(i-1)m0}\, k_{(i-1)m}\, r_{i-1}\, K_1\big(k_{(i-1)m}\, r_{i-1}\big)\Big)\Big] = 0 \quad (56)$$

$$\sum_{m=1}^{2}\Big[\delta_i\, \bar{G}_i\,\Big(\bar{A}_{imn}\,\big(n\, I_n(k_{im}\, r_{i-1}) + k_{im}\, r_{i-1}\, I_{n+1}(k_{im}\, r_{i-1})\big) + \bar{B}_{imn}\,\big(n\, K_n(k_{im}\, r_{i-1}) - k_{im}\, r_{i-1}\, K_{n+1}(k_{im}\, r_{i-1})\big)\Big) - \delta_{i-1}\, \bar{G}_{i-1}\, \bar{A}_{(i-1)mn}\,\Big(n\, I_n\big(k_{(i-1)m}\, r_{i-1}\big) + k_{(i-1)m}\, r_{i-1}\, I_{n+1}\big(k_{(i-1)m}\, r_{i-1}\big)\Big) + \bar{B}_{(i-1)mn}\,\Big(n\, K_n\big(k_{(i-1)m}\, r_{i-1}\big) - k_{(i-1)m}\, r_{i-1}\, K_{n+1}\big(k_{(i-1)m}\, r_{i-1}\big)\Big)\Big] + \bar{C}_{in}(s)\, I_n\Big(\sqrt{\frac{\psi_i\, s}{\bar{G}_{i2}}}\; r_{i-1}\Big) + \bar{D}_{in}(s)\, K_n\Big(\sqrt{\frac{\psi_i\, s}{\bar{G}_{i2}}}\; r_{i-1}\Big) - \bar{C}_{(i-1)n}(s)\, I_n\Big(\sqrt{\frac{\psi_{i-1}\, s}{\bar{G}_{(i-1)2}}}\; r_{i-1}\Big) - \bar{D}_{(i-1)n}(s)\, K_n\Big(\sqrt{\frac{\psi_{i-1}\, s}{\bar{G}_{(i-1)2}}}\; r_{i-1}\Big) = 0\,, \quad n \geq 1\,, \quad (57)$$

$$\sum_{m=1}^{2}\Big[\delta_i\, \bar{G}_i\, \frac{n}{r_{i-1}}\,\big(\bar{A}_{imn}\, I_n(k_{im}\, r_{i-1}) + \bar{B}_{imn}\, K_n(k_{im}\, r_{i-1})\big) - \delta_{i-1}\, \bar{G}_{i-1}\, \frac{n}{r_{i-1}}\,\Big(\bar{A}_{(i-1)mn}\, I_n\big(k_{(i-1)m}\, r_{i-1}\big) + \bar{B}_{(i-1)mn}\, K_n\big(k_{(i-1)m}\, r_{i-1}\big)\Big)\Big] + \bar{C}_{in}\,\Big(\frac{1}{r_{i-1}}\, I_n\Big(\sqrt{\frac{\psi_i\, s}{\bar{G}_{i2}}}\; r_{i-1}\Big) + \frac{1}{n}\sqrt{\frac{\psi_i\, s}{\bar{G}_{i2}}}\, I_{n+1}\Big(\sqrt{\frac{\psi_i\, s}{\bar{G}_{i2}}}\; r_{i-1}\Big)\Big) + \bar{D}_{in}\,\Big(\frac{1}{r_{i-1}}\, K_n\Big(\sqrt{\frac{\psi_i\, s}{\bar{G}_{i2}}}\; r_{i-1}\Big) - \frac{1}{n}\sqrt{\frac{\psi_i\, s}{\bar{G}_{i2}}}\, K_{n+1}\Big(\sqrt{\frac{\psi_i\, s}{\bar{G}_{i2}}}\; r_{i-1}\Big)\Big) - \bar{C}_{(i-1)n}\,\Big(\frac{1}{r_{i-1}}\, I_n\Big(\sqrt{\frac{\psi_{i-1}\, s}{\bar{G}_{(i-1)2}}}\; r_{i-1}\Big) + \frac{1}{n}\sqrt{\frac{\psi_{i-1}\, s}{\bar{G}_{(i-1)2}}}\, I_{n+1}\Big(\sqrt{\frac{\psi_{i-1}\, s}{\bar{G}_{(i-1)2}}}\; r_{i-1}\Big)\Big) - \bar{D}_{(i-1)n}\,\Big(\frac{1}{r_{i-1}}\, K_n\Big(\sqrt{\frac{\psi_{i-1}\, s}{\bar{G}_{(i-1)2}}}\; r_{i-1}\Big) - \frac{1}{n}\sqrt{\frac{\psi_{i-1}\, s}{\bar{G}_{(i-1)2}}}\, K_{n+1}\Big(\sqrt{\frac{\psi_{i-1}\, s}{\bar{G}_{(i-1)2}}}\; r_{i-1}\Big)\Big) = 0\,, \qquad n \geq 1\,, \quad (58)$$

$$\sum_{m=1}^{2}\Big[\delta_i\, \bar{G}_i\, \bar{G}_{i2}\,\Big(\bar{A}_{im0}\,\Big(\Big(\frac{r_{i-1}^2\, \psi_i\, s}{2\, \bar{G}_{i2}}\Big)\, I_0(k_{im}\, r_{i-1}) - k_{im}\, r_{i-1}\, I_1(k_{im}\, r_{i-1})\Big) + \bar{B}_{im0}\,\Big(\Big(\frac{r_{i-1}^2\, \psi_i\, s}{2\, \bar{G}_{i2}}\Big)\, K_0(k_{im}\, r_{i-1}) + k_{im}\, r_{i-1}\, K_1(k_{im}\, r_{i-1})\Big)\Big) - \delta_{i-1}\, \bar{G}_{i-1}\, \bar{G}_{(i-1)2}\,\Big(\bar{A}_{(i-1)m0}\,\Big(\Big(\frac{r_{i-1}^2\, \psi_{i-1}\, s}{2\, \bar{G}_{(i-1)2}}\Big)\, I_0\big(k_{(i-1)m}\, r_{i-1}\big) - k_{(i-1)m}\, r_{i-1}\, I_1\big(k_{(i-1)m}\, r_{i-1}\big)\Big) + \bar{B}_{(i-1)m0}\,\Big(\Big(\frac{r_{i-1}^2\, \psi_{i-1}\, s}{2\, \bar{G}_{(i-1)2}}\Big)\, K_0\big(k_{(i-1)m}\, r_{i-1}\big) + k_{(i-1)m}\, r_{i-1}\, K_1\big(k_{(i-1)m}\, r_{i-1}\big)\Big)\Big)\Big] = \frac{r_{i-1}^2\, Q_0}{2\, s^2}\begin{cases}\delta_i\, \gamma_i\, \bar{G}_i - \delta_{i-1}\, \gamma_{i-1}\, \bar{G}_{i-1} & , \text{ for } \quad i = 1, 2, 3, \cdots, w \\ -\delta_{i-1}\, \gamma_{i-1}\, \bar{G}_{i-1} & , \text{ for } \quad i = w+1 \\ 0 & , \text{ for } \quad i = w+2, w+3, \cdots, k\end{cases}, \quad (59)$$

$$\sum_{m=1}^{2}\Big[\delta_i\, \bar{G}_i\, \bar{G}_{i2}\,\Big(\bar{A}_{imn}\,\Big(\Big(n(n-1) + \frac{r_{i-1}^2\, \psi_i\, s}{2\, \bar{G}_{i2}}\Big)\, I_n(k_{im}\, r_{i-1}) - k_{im}\, r_{i-1}\, I_{n+1}(k_{im}\, r_{i-1})\Big) + \bar{B}_{imn}\,\Big(\Big(n(n-1) + \frac{r_{i-1}^2\, \psi_i\, s}{2\, \bar{G}_{i2}}\Big)\, K_n(k_{im}\, r_{i-1}) + k_{im}\, r_{i-1}\, K_{n+1}(k_{im}\, r_{i-1})\Big)\Big) - \delta_{i-1}\, \bar{G}_{i-1}\, \bar{G}_{(i-1)2}\,\Big(\bar{A}_{(i-1)mn}\,\Big(\Big(n(n-1) + \frac{r_{i-1}^2\, \psi_{i-1}\, s}{2\, \bar{G}_{(i-1)2}}\Big)\, I_n\big(k_{(i-1)m}\, r_{i-1}\big) - k_{(i-1)m}\, r_{i-1}\, I_{n+1}\big(k_{(i-1)m}\, r_{i-1}\big)\Big) + \bar{B}_{(i-1)mn}\,\Big(\Big(n(n-1) +$$

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$$\frac{r_{i-1}^2\,\psi_{i-1}\,s}{2\,\bar{G}_{(i-1)2}}\Big)\,K_n\big(k_{(i-1)m}\,r_{i-1}\big)+k_{(i-1)m}\,r_{i-1}\,K_{n+1}\big(k_{(i-1)m}\,r_{i-1}\big)\Big)\Big]+\bar{G}_{i2}\,\bar{C}_{in}\left((n-1)\,I_n\left(\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r_{i-1}\right)+\right.$$

$$\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r_{i-1}\,I_{n+1}\left(\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r_{i-1}\right)\Bigg)+\bar{G}_{i2}\,\bar{D}_{in}\left((n-1)\,K_n\left(\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r_{i-1}\right)-\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r_{i-1}\,K_{n+1}\left(\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r_{i-1}\right)\right)-$$

$$\bar{G}_{(i-1)2}\,\bar{C}_{(i-1)n}\left((n-1)\,I_n\left(\sqrt{\frac{\psi_{i-1}\,s}{\bar{G}_{(i-1)2}}}\;r_{i-1}\right)+\sqrt{\frac{\psi_{i-1}\,s}{\bar{G}_{(i-1)2}}}\;r_{i-1}\,I_{n+1}\left(\sqrt{\frac{\psi_{i-1}\,s}{\bar{G}_{(i-1)2}}}\;r_{i-1}\right)\right)-\bar{G}_{(i-1)2}\,\bar{D}_{(i-1)n}\Bigg((n-$$

$$1)\,K_n\left(\sqrt{\frac{\psi_{i-1}\,s}{\bar{G}_{(i-1)2}}}\;r_{i-1}\right)-\sqrt{\frac{\psi_{i-1}\,s}{\bar{G}_{(i-1)2}}}\;r_{i-1}\,K_{n+1}\left(\sqrt{\frac{\psi_{i-1}\,s}{\bar{G}_{(i-1)2}}}\;r_{i-1}\right)\Bigg)=$$

$$\frac{r_{i-1}^2\,Q_0}{2\,s^2}\begin{cases}\delta_i\,\gamma_i\,\bar{G}_i-\delta_{i-1}\,\gamma_{i-1}\,\bar{G}_{i-1} & ,\ \text{for}\quad i=1,2,3,\cdots,w\\ -\delta_{i-1}\,\gamma_{i-1}\,\bar{G}_{i-1} & ,\ \text{for}\quad i=w+1\\ 0 & ,\ \text{for}\quad i=w+2,w+3,\cdots,k\end{cases}\quad,\quad n\ge 1\,, \tag{60}$$

$$\sum_{m=1}^{2}\Big[\delta_i\,\bar{G}_i\,\bar{G}_{i2}\,\Big(\bar{A}_{imn}\,n\,\big((n-1)\,I_n(k_{im}\,r_{i-1})+k_{im}\,r_{i-1}\,I_{n+1}(k_{im}\,r_{i-1})\big)+\bar{B}_{imn}\,n\,\big((n-$$

$$1)\,K_n(k_{im}\,r_{i-1})-k_{im}\,r_{i-1}\,K_{n+1}(k_{im}\,r_{i-1})\big)\Big)-\delta_{i-1}\,\bar{G}_{i-1}\,\bar{G}_{(i-1)2}\,\Big(\bar{A}_{(i-1)mn}\,n\,\big((n-1)\,I_n\big(k_{(i-1)m}\,r_{i-1}\big)+$$

$$k_{(i-1)m}\,r_{i-1}\,I_{n+1}\big(k_{(i-1)m}\,r_{i-1}\big)\big)+\bar{B}_{(i-1)mn}\,n\,\big((n-1)\,K_n\big(k_{(i-1)m}\,r_{i-1}\big)-k_{(i-1)m}\,r_{i-1}\,K_{n+1}\big(k_{(i-1)m}\,r_{i-1}\big)\big)\Big)\Big]+$$

$$\frac{\bar{G}_{i2}}{n}\Bigg[\bar{C}_{in}\left(\left(n\,(n-1)+\frac{r_{i-1}^2\,\psi_i\,s}{2\,\bar{G}_{i2}}\right)I_n\left(\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r_{i-1}\right)-\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r_{i-1}\;I_{n+1}\left(\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r_{i-1}\right)\right)+\bar{D}_{in}\Bigg(\Big(n\,(n-1)+$$

$$\frac{r_{i-1}^2\,\psi_i\,s}{2\,\bar{G}_{i2}}\Big)\,K_n\left(\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r_{i-1}\right)+\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r_{i-1}\;K_{n+1}\left(\sqrt{\frac{\psi_i\,s}{\bar{G}_{i2}}}\;r_{i-1}\right)\Bigg)\Bigg]-\frac{\bar{G}_{(i-1)2}}{n}\Bigg[\bar{C}_{(i-1)n}\Bigg(\Big(n\,(n-1)+$$

$$\frac{r_{i-1}^2\,\psi_{i-1}\,s}{2\,\bar{G}_{(i-1)2}}\Big)\,I_n\left(\sqrt{\frac{\psi_{i-1}\,s}{\bar{G}_{(i-1)2}}}\;r_{i-1}\right)-\sqrt{\frac{\psi_{i-1}\,s}{\bar{G}_{(i-1)2}}}\;r_{i-1}\;I_{n+1}\left(\sqrt{\frac{\psi_{i-1}\,s}{\bar{G}_{(i-1)2}}}\;r_{i-1}\right)\Bigg)+\bar{D}_{(i-1)n}\Bigg(\Big(n\,(n-1)+$$

$$\frac{r_{i-1}^2\,\psi_{i-1}\,s}{2\,\bar{G}_{(i-1)2}}\Big)\,K_n\left(\sqrt{\frac{\psi_{i-1}\,s}{\bar{G}_{(i-1)2}}}\;r_{i-1}\right)+\sqrt{\frac{\psi_{i-1}\,s}{\bar{G}_{(i-1)2}}}\;r_{i-1}\;K_{n+1}\left(\sqrt{\frac{\psi_{i-1}\,s}{\bar{G}_{(i-1)2}}}\;r_{i-1}\right)\Bigg)\Bigg]=0\,,\quad n\ge 1\,, \tag{61}$$

By solving the system (49) - (61), we obtain the unknowns $\bar{A}_{imn}(s)$, $\bar{B}_{imn}(s)$, $\bar{C}_{in}(s)$ and $\bar{D}_{in}(s)$.

IV. INVERSION OF THE LAPLACE TRANSFORM

We can apply a numerical method based on Fourier expansions of functions to invert the Laplace transform. The Laplace transform's inversion formula for a function $\bar{f}(x,s)$ can be expressed as follows:

$$f(x,t)=\frac{1}{2\pi i}\int_{\gamma-j\,\infty}^{\gamma+j\,\infty}e^{s\,t}\,\bar{f}(x,s)\,ds\,, \tag{62}$$

where γ is an arbitrary real number greater than all the real parts of the singularities of $\bar{f}(x,s)$. We obtain approximate formula for (62) by considering $s=\gamma+j\,y$ and the Fourier series in the range $[0,2L]$.

$$f_N(x,t)=\frac{1}{2}\,c_0+\sum_{k=1}^{N}c_k\,,\quad for\quad 0\le t\le 2\,L\,, \tag{63}$$

where

$$c_k=\frac{e^{\gamma\,t}}{L}\,Re\,[\,e^{j\,k\,\pi\,t/L}\,\bar{f}(x,\gamma+j\,k\,\pi/L)\,]\,. \tag{64}$$

To decrease the overall inaccuracy, two techniques are applied. First, the discretization error is reduced using the Korrecktur approach. The ε-algorithm is then applied to minimize the truncation error and speed up convergence, [6].

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V. NUMERICAL RESULTS AND DISCUSSION

In this part, we use the previous numerical technique to illustrate the physical behavior of the quantities, temperature, radial displacement and radial stress inside a multilayered cylinder defined in Section 2, in which the number of layers $k = 4$. These quantities are investigated in terms of how they change over time t, under varies of empirical parameters A_1, A_2 and the strength of the heat source Q_0. Considering, 1. The surrounding temperature

$$\boldsymbol{F(\varphi, t) = \theta_0\, H(t)\, H(\varphi_0 - |\varphi|)}\,, \tag{65}$$

where θ_0 represents the strength of the temperature on the boundary, $H(t)$ is the Heaviside function and $\varphi_0 = \pi/12$ during computations. This means that the cylinder surface is subjected to a thermal shock strength θ_0 over the sector $-\varphi_0 \leq \varphi \leq \varphi_0$ and zero everywhere else. Thermal shock is a one-time, uneven rapid change in the temperature of a body in the order of tens or hundreds of degrees per sec, which usually refers to cases of rapid heating. The main factors in thermal shock are the rapid development of a temperature gradient in the order of fractions of a second, the appearance of deformations and stresses, causing a change in shape, formation of cracks or destruction of a body.

2. The relaxation functions

$$\boldsymbol{G_{i1}(t) = \lambda_i\,(1 - A_1\, e^{-\gamma t}) \quad and \quad G_{i2}(t) = \mu_i\,(1 - A_2\, e^{-\gamma t})} \tag{66}$$

where A_1, A_2 and γ are empirical constants such that $\gamma \geq 0$. When stress reaching it's peak under a constant strain, we refer to it's behaviour to decrease or relax over time as stress relaxation, which is considered as one of the main characteristic of viscoelastic materials. In comparison, elastic materials do not undergo such behaviour, it reaches a fixed stress and stays at that level with no relaxation. Set the empirical parameters $A_{i1} = A_{i2} = 0$ to model the i_{th} layer as elastic material.

Considering the materials of the cylinder layers made of polymethyl methacrylate (PMMA) and copper for the case of numerical solution of the problem. Reference temperature and relaxation time for both materials are taken as $T_0 = 293\ K$ and $\tau = 0.02\ s$, respectively. The problem's constants (in SI units) are presented in Table 1, see Sherief et al. [15] for the copper material and Sherief and Allam [13] for PMMA material.

Copper	**Physical Properties**	$\rho = 8954\ kg/m^3$		
	Mechanical Properties	$\lambda = 7.76\,(10)^{10}\ kg/(m\,s^2)$	$\mu = 3.86\,(10)^{10}\ kg/(m\,s^2)$	
	Thermal Properties	$\alpha = 1.78\,(10)^{-5}\ K^{-1}$	$c = 383.1\ J/(kg\ K)$	$k = 386\ W/(m\ K)$
PMMA	**Physical Properties**	$\rho = 1160\ kg/m^3$		
	Mechanical Properties	$\lambda = 0.4\,(10)^{10}\ kg/(m\,s^2)$	$\mu = 0.19\,(10)^{10}\ kg/(m\,s^2)$	
	Thermal Properties	$\alpha = 6.3\,(10)^{-5}\ K^{-1}$	$c = 1475\ J/(kg\ K)$	$k = 187\ W/(m\ K)$

Table 1: PMMA and Copper materials constants

Physical properties of the considered model are studied along the polar line ($\varphi = 0$ and $\varphi = \pi$). Along this line, the only non vanishing components for displacement vector u and stress tensor σ are the radial displacement u_r and radial normal stress σ_{rr}, respectively. Wave propagation for these components and temperature are studied under the evolution of time in Figure 1 and 2 for the two cases $Q_0 = 0.5$ and $Q_0 = 0.0$ (In the absence of a heat source), respectively.

Figures 1(a) and 2(a), show a propagation of two wave fronts from the outer surface of the cylinder and heading towards the interior, having a mechanical and thermal nature respectively. Clearly the thermal wave front is dominant while the mechanical has no significant effect. A close case is represented in Figures 1(b) and 2(b) where the longitudinal elastic (mechanical) wave transform the head of the wave into cusp-shaped, while the thermal wave has a negligible effect. For example, at $t = 0.2$, the two wave fronts are located at $r = 0.9688$ and $r = 0.6408$.

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In addition, from Figures 1(b) and 2(b), an expansion in the outermost and innermost layers of the cylinder due to the heating acting on the boundary and the heat source acting on the inner layers, whereas the unheated intermediate layers seem to be resisting expansion. Moreover, with the passage of time, the solid acquires more and more amounts of heat that leads to an increase in the expanded layers and a decrease in the compressed ones. Furthermore, a tensile stress, the resistance of a material or construction to loads that tend to elongate in size, arises in the more heated layers and a compressive stress, the ability of a substance or structure to tolerate loads that tend to cause it to shrink in size, arises in the less heated layers, see Figures 1(c) and 2(c).

The effect of the empirical constants A_1 and A_2 on the wave propagation for temperature, radial displacement and normal stress is studied in Figure 3. It is evident that any change in the parameters A_1 and A_2 has a negligible effect on the temperature, whereas a reduction in either of these two parameters will result in a reduction in the number of compressed layers as well as an increase in the number of expanded layers. Thus we can assume that empirical parameters have a no significant effect on the thermal properties (temperature) but effect the mechanical properties (displacement and stress). In addition to this, they have an effect on the layers that are exposed to the thermal shock on the cylinder's border, not the layers subjected to the internal heat source.

Figure 4 illustrates the distributions of the problem against different strengths of the heat source Q_0. It can be seen that the magnitude of all the physical properties, temperature, radial displacement and radial stress, are proportional to the strength of the heat source. In addition to this, it is evident that in the absence of an internal heat source, all of the distributions taken into consideration have a value were taken into consideration have a value that is greater than zero only inside a restricted region and disappears in the same way outside of this region, which is in agreement with the idea of the second sound effect, every possible solution to the problem has a finite speed of thermal disturbance, which was justified by [3]. From Figure 1, it can be seen that this region expands with time.

VI. CONCULSION

In this work, the effects of a thermal shock operating on the boundary and a dispersed heat source acting within the medium are examined in relation to the thermal and elastic properties of a solid in the shape of an indefinitely long circular cylinder made up of k viscoelastic and/or elastic layers. The Laplace transform approach is used to produce an analytical solution for the temperature, displacement vector, and stress tensor. Based on our research, we have come to the following conclusions:

a) Thermal and elastic waves propagate with finite speed of thermal disturbance satisfying the concept of the second sound effect.

b) Physical properties have mechanical and thermal jump that propagate from the surface of the cylinder to its interior.

c) The more heated layers, the outer layers subjected to the thermal shock and the most inner layers subjected to the heat source, suffer from tensile stress whereas the less heated layers suffer from compressive stress.

d) The empirical parameters, describing the stress relaxation functions, affect the mechanical properties, displacement and stress, while having negligible effect on the temperature.

e) The magnitude of all the physical properties, temperature, radial displacement and radial stress, are proportional to how much heat the heat source produces.

f) The maximum stress occurs at its wave front due to the thermal shock on the boundary or at the center of the cylinder due to the heat source.

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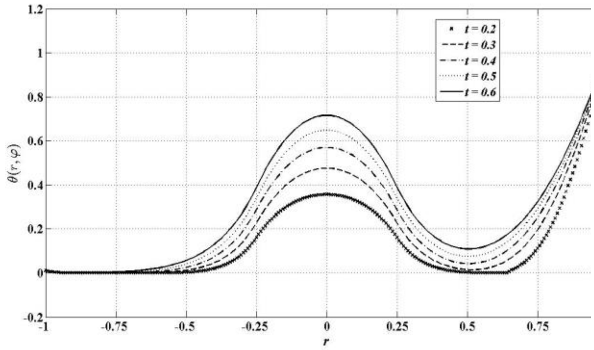
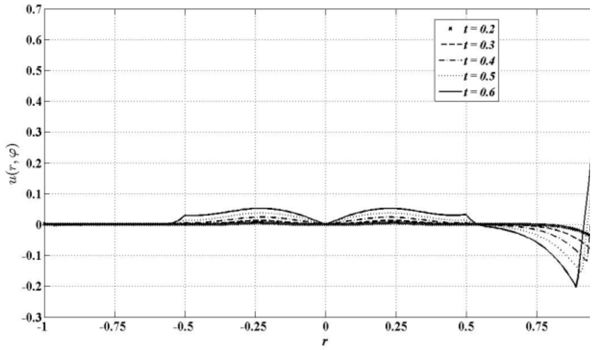
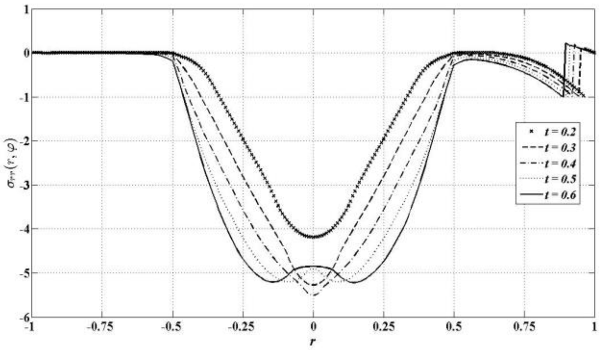


(a) Temperature

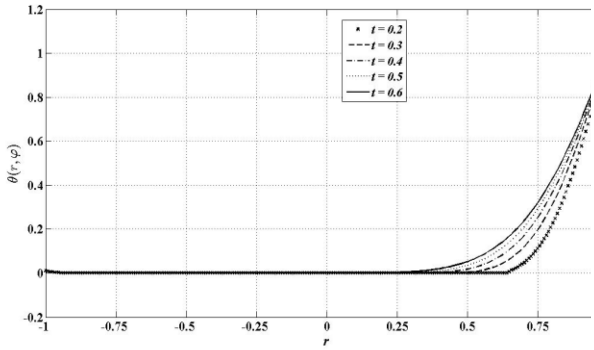


(b) Displacement

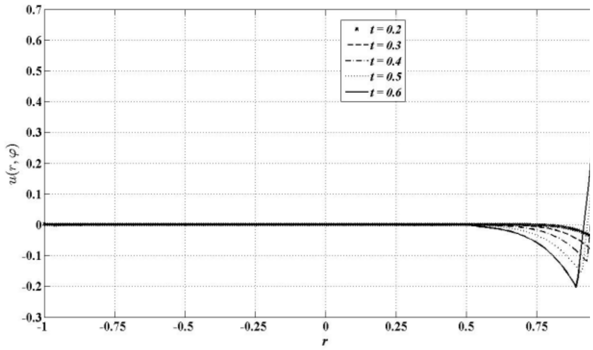


(c) Stress

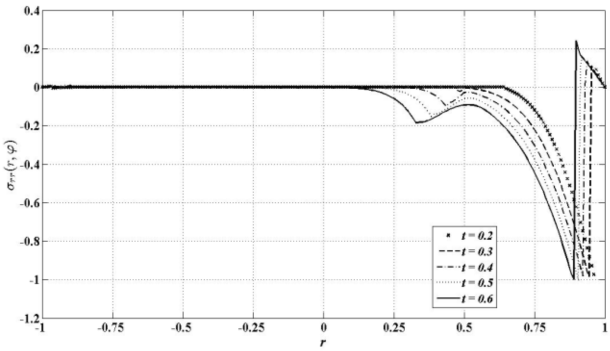
Figure 1: Radial distributions for times t at $A_1 = A_2 = 0.5$, $\gamma = 0.5$, $\omega = 1$ and $Q_0 = 1$.



(a) Temperature



(b) Displacement



(c) Stress

Figure 2: Radial distributions for times t at $A_1 = A_2 = 0.5$, $\gamma = 0.5$, $\omega = 1$ and $Q_0 = 0$.

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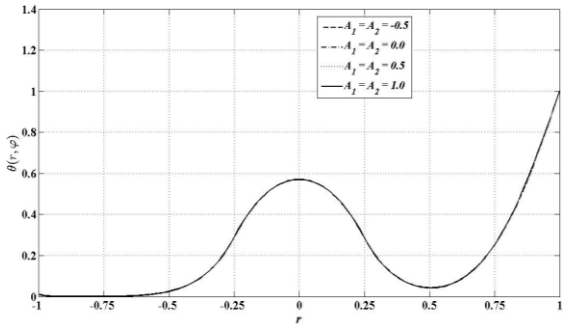
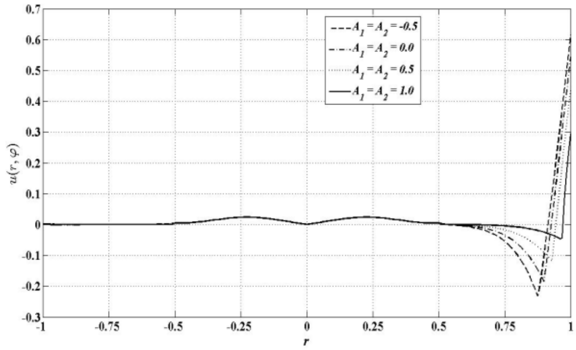
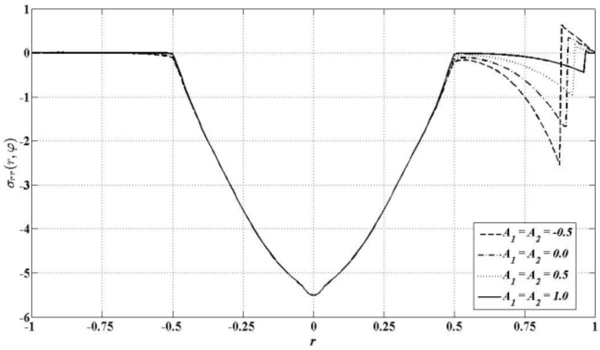


(a) Temperature

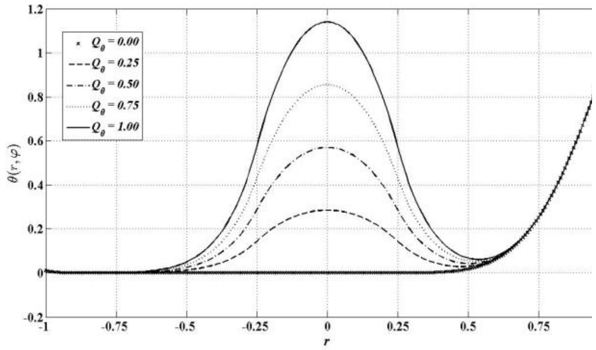


(b) Displacement

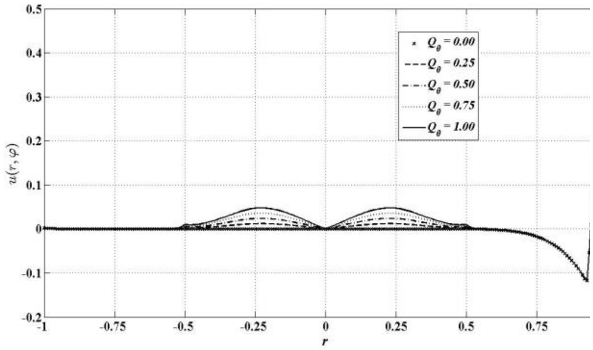


(c) Stress

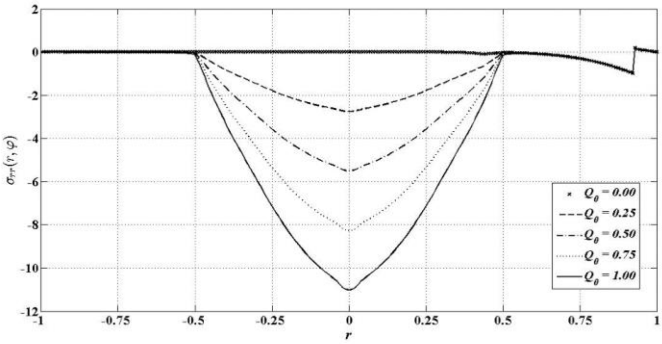
Figure 3: Radial distributions for different values of the empirical constants A_1 and A_2 at $t = 0.4,\ \gamma = 0.5,\ \omega = 1$ and $Q_0 = 1$.



(a) Temperature



(b) Displacement



(c) Stress

Figure 4: Radial distributions for different values of the strength of the induced heat source Q_0 at $t = 0.4,\ \gamma = 0.5,\ \omega = 1$ and $A_1 = A_2 = 0.5$.

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